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Compact Model and System Simulation of a Machine Tool

INTRODUCTION AND MOTIVATION

The stability and repeatability of a machining process are strongly impacted by thermal effects, which emerge from internal heat sources, like electrical drives, or friction between the tool and the workpiece.

Numerical simulation offers the possibility for systematic monitoring and real-time compensation of thermally induced deformations and displacements of a machine tool. For developing an adequate positioning controller, a reduced-order numerical model is required.

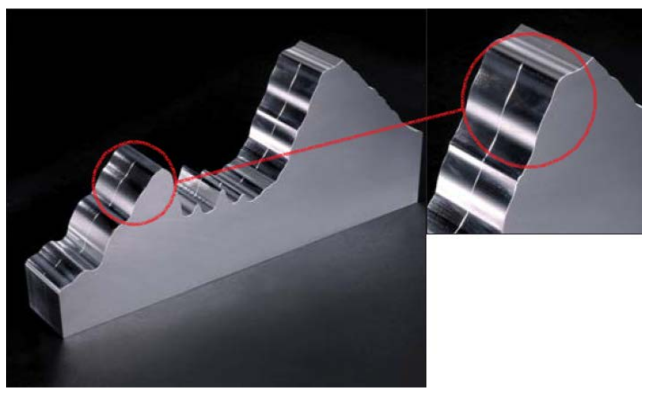


Fig 1. Thermally induced deformations [1]

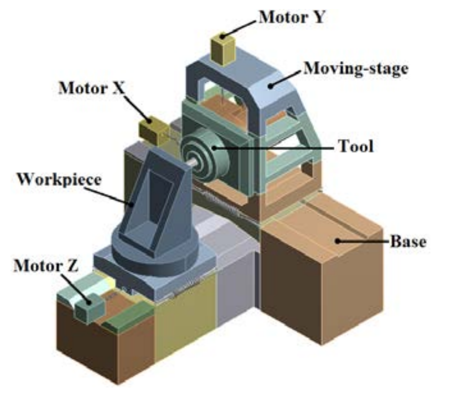


Fig 2. Structure of the machine tool model

MODEL ORDER REDUCTION

A strongly coupled thermo-mechanical machine tool model was built in ANSYS Mechanical®. Spatial discretization by finite element method (FEM) yields a large system of 75.729 second order ordinary differential equations (ODEs) of the form:

$$\Sigma_N : \begin{cases} \begin{bmatrix} M_U & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{T} \end{Bmatrix} + \begin{bmatrix} E_U & 0 \\ E^{tu} & E_T \end{bmatrix} \begin{Bmatrix} \dot{U} \\ \dot{T} \end{Bmatrix} + \begin{bmatrix} K_U & K^{ut} \\ 0 & K_T \end{bmatrix} \begin{Bmatrix} U \\ T \end{Bmatrix} = B \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \\ y = C \begin{Bmatrix} U \\ T \end{Bmatrix} \end{cases}$$

As time integration of $\sum_N$ is computationally extensive, a compact but highly accurate numerical model of the same form is generated by mathematical model order reduction [2]. In particular we use the Krylov-subspace based Second Order Arnoldi Reduction algorithm (SOAR) from [3]. Furthermore, a Schur-Complement transformation, as suggested in [4], is implemented to transfer the model into state-space form, which is required for the system-level simulation in ANSYS Simplorer®.

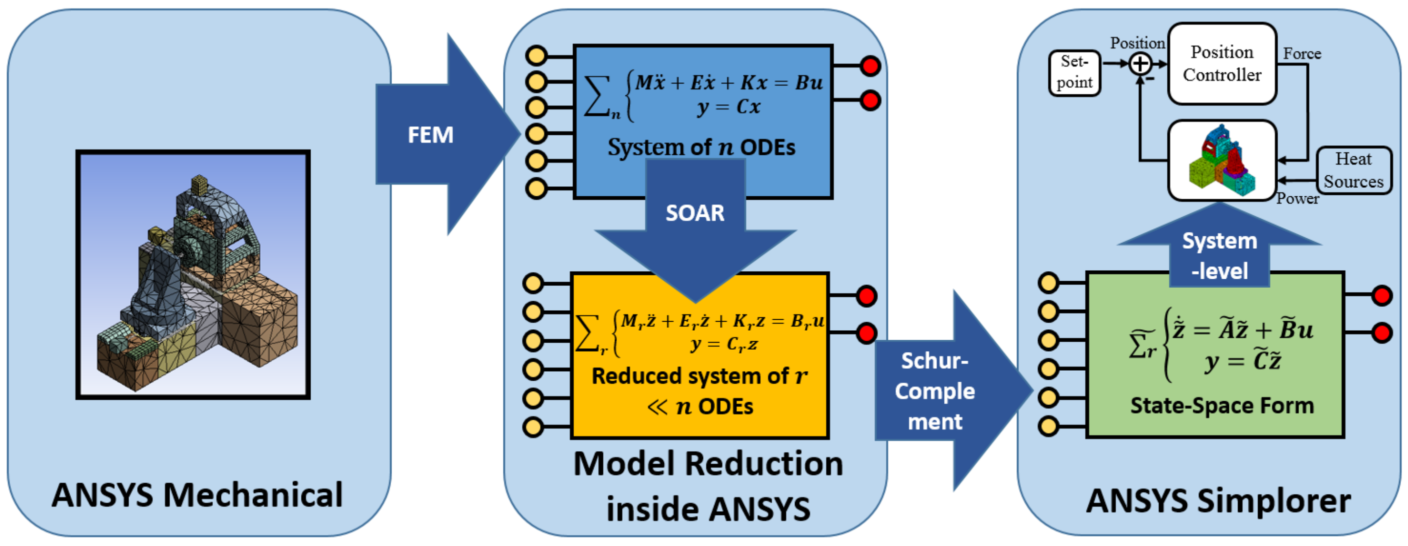


Fig 3. Workflow for the system-level simulation of the machine tool model

NUMERICAL MODELLING AND RESULTS

The thermo-mechanical model $\sum_N$ has two inputs: force applied onto the moving-stage and heat generation applied at the spindle of the tool, and two outputs: displacement and temperature at the tool center point. Hence, four transfer functions can be defined, G_{11} = displacement output/force input, G_{12} = temperature/force input, G_{21} = displacement output/heat input, G_{22} = temperature output/heat input.

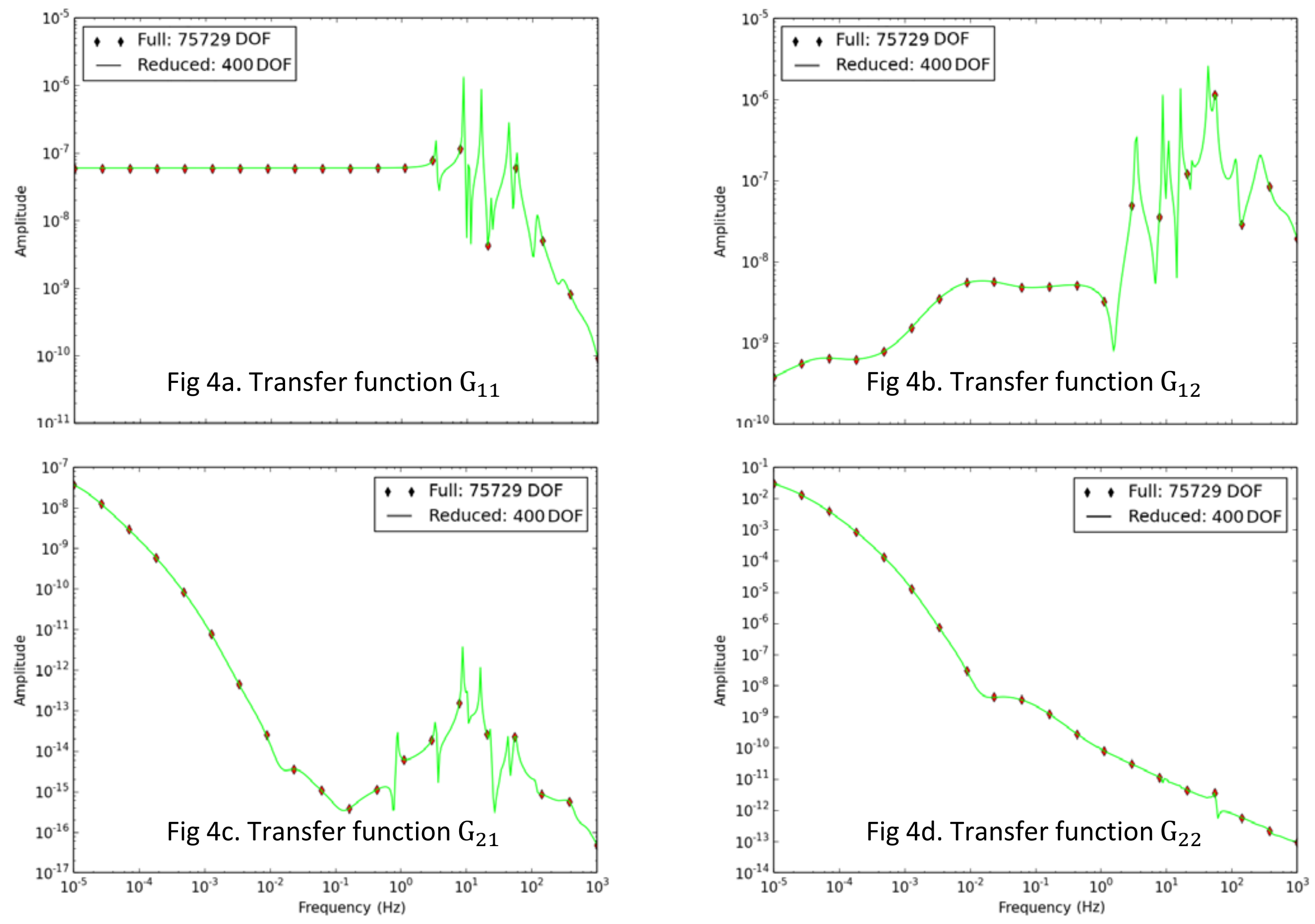


Fig 4a. Transfer function G_{11}

Fig 4b. Transfer function G_{12}

Fig 4c. Transfer function G_{21}

Fig 4d. Transfer function G_{22}

The strongly coupled machine tool model suffers from different time constants for the structural dynamics and heat transfer behavior. The reduced model has to match the frequency range from 0Hz to 10Hz (thermal expansion) as well as the range from 1Hz to 1000Hz (dynamics).

The convergence study for different reduced model dimensions is conducted. The relative error between the full and reduced model's transfer function is defined as:

$$\varepsilon = \frac{\|G(s) - G_r(s)\|_2}{\|G(s)\|_2}$$

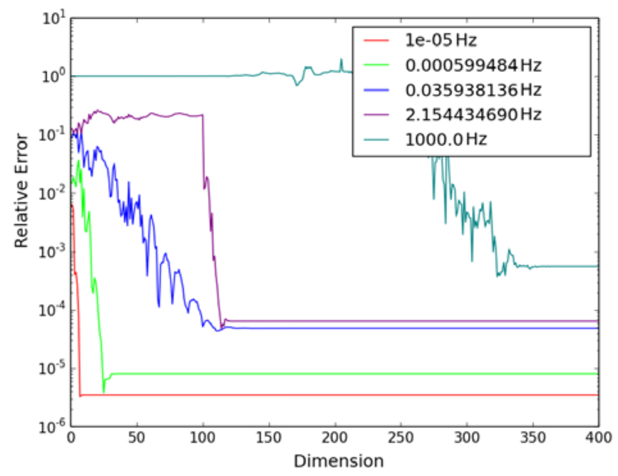


Fig 5. Reduced model with only force input

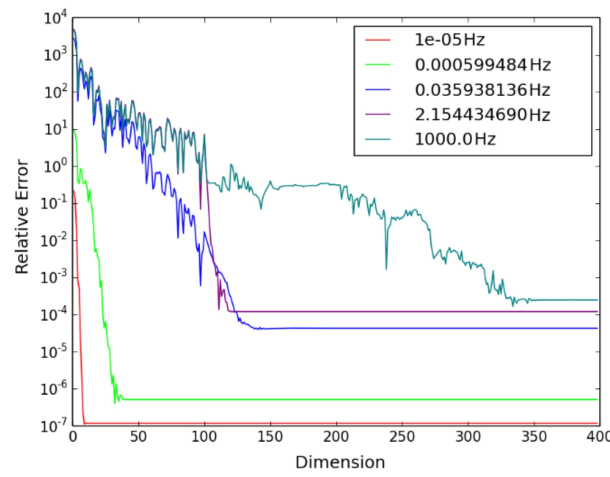
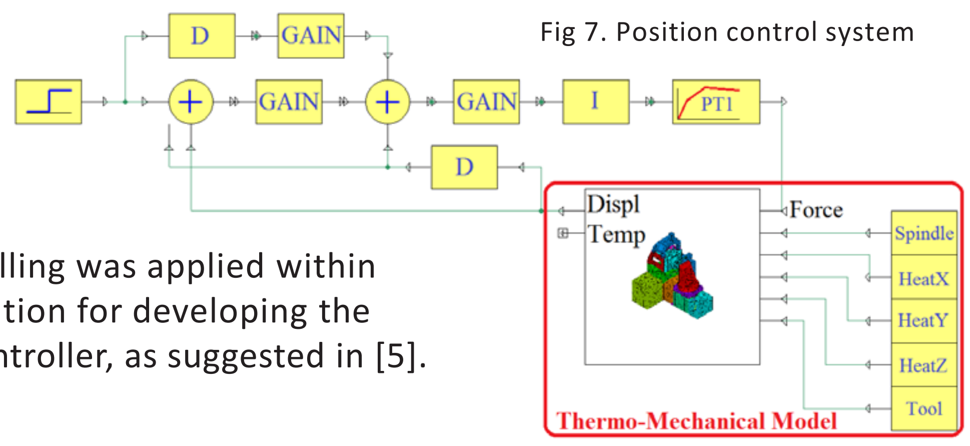


Fig 6. Reduced model with only heat input

The convergence plots above show the relative error versus reduced models' dimensions at different frequencies. In order to match both relevant frequency ranges, an accurate reduced thermo-mechanical model (with relative error of 0.1%) should have at least the dimension 350.

Fig 7. Position control system



Reduced order modelling was applied within a system-level simulation for developing the real-time position controller, as suggested in [5].

CONCLUSIONS

- We demonstrated that the reduced order model, created by mathematical Krylov-subspace based model order reduction, reflects all physical effects of the full-scale machine tool model.
- It could be applied within a system level simulation to develop the real-time position controller.

LITERATURE

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